

# Strong coupling theory for the superfluidity of Bose-Fermi mixtures

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We develop a strong coupling theory for the superfluidity of fermion pairing phase in a Bose-Fermi mixture. Dynamical screening, self-energy renormalization, and pairing gap function are included self-consistently within the adiabatic limit (i.e. the phonon velocity is much smaller than the Fermi velocity). An analytical solution for the transition temperature ( $T_c$ ) is derived within reasonable approximations. Using typical parameters of a  $^{40}\text{K}$ - $^{87}\text{Rb}$  mixture, we find that the calculated  $T_c$  is several times larger than that obtained in the weak coupling theory, and can be up to several percents of the Fermi temperature.

Recently superfluidity of fermion pairing phase has become an extensively studied subject in the field of ultracold atoms [1,2,3,4,5,6,7]. The attractive interaction for two fermions to form a Cooper pair can be provided either by the direct  $s$ -wave scattering between different spin states [1,2,3], or by the effective interaction mediated by the condensate phonons in a Bose-Fermi mixture (BFM) [4,5]. The former systems attract a lot of attention on the BCS-BEC crossover near the Feshbach resonance regime [3], while the latter systems, which can mimic some traditional electron-phonon systems in solid state physics, are expected to have many interesting many-body phases in different parameter regimes [6,7,8].

To observe the predicted fermion pairing phase or other many-body states in a BFM, experimentalists usually have to increase the boson-fermion interaction strength via a heteronuclear Feshbach resonance [7,9]. However, by doing so the system will be driven into a strong coupling regime, where the direct application of the Bardeen-Cooper-Schrieffer (BCS) theory for the superfluidity of fermion pairing phase becomes unjustified and not reliable. For example, the strong fermion-boson interaction may renormalize the effective mass of fermions and hence change its density of states near the Fermi surface. Besides, the fermionic quasi-particle fluctuations at finite temperature can also dynamically screen the effective interaction induced by condensate phonons. None of these effects have been included in the weak coupling theory developed in the literature [4,5,6]. Moreover, in most of the weak coupling theories, one usually applies an instantaneous approximation (i.e. assuming the phonon velocity,  $c_b$ , is much larger than the Fermi velocity,  $v_F$ ). Such approximation is, however, very unrealistic in typical systems like  $^{40}\text{K}$ - $^{87}\text{Rb}$  or  $^6\text{Li}$ - $^{23}\text{Na}$  mixtures, which are usually in the adiabatic regime ( $c_b/v_F \sim 0.05 - 0.5 < 1$ ) due to the weak interaction between bosonic atoms [5]. Therefore in order to have a reasonable comparison between the theoretical results and the experimental measurement, it is necessary and important to develop a full strong coupling theory, including the adiabaticity of condensate phonons as well as the strong correlation effects of a BFM self-consistently.

In this Letter, we develop a strong coupling theory for the superfluidity of ( $s$ -wave) fermion pairing phase in a BFM within the adiabatic regime by generalizing the cel-

ebrated Migdal-Eliashberg equations used in the conventional superconductors [10,11]. By strong coupling, we mean to treat the fermion self-energy, dynamical screening of the effective interaction, and the pairing gap function self-consistently. Hence the obtained results are reliable even when the fermion-boson interaction is not weak. Vertex diagrams are safely neglected by applying Migdal's theorem in the adiabatic regime of phonons [10,11]. Using a single mode approximation we further derive an analytical solution for the superfluidity transition temperatures and compare them with the known weak coupling BCS results in different parameter regime. In a typical  $^{40}\text{K}$ - $^{87}\text{Rb}$  mixture, we find that the obtained  $T_c$  is several times larger than the weak coupling results, and can be up to a few percents of the Fermi energy before reaching phase instability. Effects due to strong boson-fermion correlations are also critically discussed.

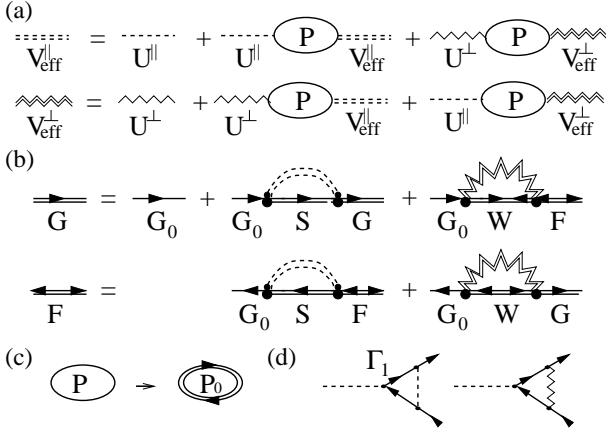
We consider a three dimensional BFM composed of spin polarized bosons and fermions in two equally occupied hyperfine spin states. For simplicity, we neglect the inhomogeneous magnetic trap potential so that the total Hamiltonian can be written to be [6]

$$H = \sum_{\mathbf{k}} \left[ \epsilon_{\mathbf{k}}^b \hat{b}_{\mathbf{k}}^\dagger \hat{b}_{\mathbf{k}} + \epsilon_{\mathbf{k},\uparrow}^f \hat{f}_{\mathbf{k},\uparrow}^\dagger \hat{f}_{\mathbf{k},\uparrow} + \epsilon_{\mathbf{k},\downarrow}^f \hat{f}_{\mathbf{k},\downarrow}^\dagger \hat{f}_{\mathbf{k},\downarrow} \right] \quad (1)$$

$$+ \frac{1}{\Omega} \sum_{\mathbf{k}} \left[ \frac{U_{bb}}{2} \hat{\rho}_{\mathbf{k}}^b \hat{\rho}_{-\mathbf{k}}^b + U_{bf} \hat{\rho}_{\mathbf{k}}^b \hat{\rho}_{-\mathbf{k}}^f + U_{ff} \hat{\rho}_{\mathbf{k},\uparrow}^f \hat{\rho}_{-\mathbf{k},\downarrow}^f \right]$$

where  $\hat{b}_{\mathbf{k}}$  and  $\hat{f}_{\mathbf{k},s}$  are the field operators for bosonic and fermionic atoms of momentum  $\mathbf{k}$  and isospin  $s = \uparrow, \downarrow$ .  $\hat{\rho}_{\mathbf{k}}^b = \sum_{\mathbf{p}} \hat{b}_{\mathbf{p}+\mathbf{k}}^\dagger \hat{b}_{\mathbf{p}}$  is the boson density operator, and  $\epsilon_{\mathbf{k}}^b = \mathbf{k}^2/2m_b - \mu_b$  is the bosonic kinetic energy with  $m_b$  being the atom mass and  $\mu_b$  being the chemical potential. Similar notations also apply to fermions with super/subscript  $f$ , and  $\hat{\rho}_{\mathbf{k}}^f \equiv \hat{\rho}_{\mathbf{k},\uparrow}^f + \hat{\rho}_{\mathbf{k},\downarrow}^f$ .  $U_{bb} = \frac{4\pi a_{bb}}{m_b}$ ,  $U_{ff} = \frac{4\pi a_{ff}}{m_f}$ , and  $U_{bf} = \frac{2\pi a_{bf}}{m_r}$ , are respectively the boson-boson, fermion-fermion and boson-fermion pseudo-potential strength, with  $a_{ij}$  being the associate  $s$ -wave scattering length.  $m_r$  is the reduced mass and  $\Omega$  is the system volume.

Throughout this Letter we assume that the temperature is so low that all bosons are condensed at zero momentum state. The condensate excitations (phonons) are of Bogoliubov type dispersion:  $\omega_{\mathbf{k}}^0 = c_b |\mathbf{k}| \sqrt{1 + |\mathbf{k}|^2 \xi_b^2}$ ,

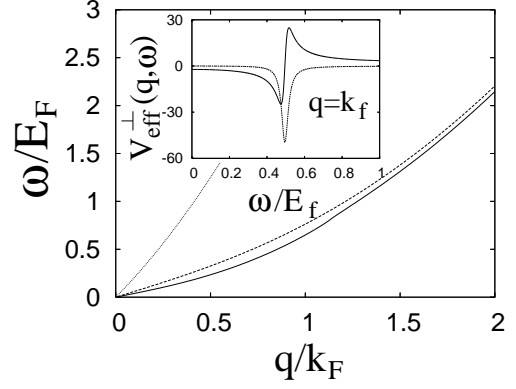


where  $c_b \equiv \sqrt{n_b U_{bb}/m_b}$  and  $\xi_b \equiv \sqrt{1/4m_b n_b U_{bb}}$  are respectively the phonon velocity and the healing length of the condensate with density  $n_b$ . As a result, one can effectively describe the BFM system by the following fermion-phonon type Hamiltonian [4,5,6]:  $H = \sum_{\mathbf{k},s} \bar{\epsilon}_{\mathbf{k},s}^f \hat{f}_{\mathbf{k},s}^\dagger \hat{f}_{\mathbf{k}} + \sum_{\mathbf{k}} \left[ \omega_{\mathbf{k}}^0 \beta_{\mathbf{k}}^\dagger \beta_{\mathbf{k}} + g_{\mathbf{k}} (\hat{\beta}_{\mathbf{k}} + \hat{\beta}_{-\mathbf{k}}^\dagger) \hat{\rho}_{-\mathbf{k}}^f \right]$  where  $\hat{\beta}_{\mathbf{k}}$  is the phonon operator and  $g_{\mathbf{k}} = U_{bf} \epsilon_{\mathbf{k}}^b / \omega_{\mathbf{k}}^0$  measures the fermion-phonon coupling strength. Integrating out the phonon field, one can obtain a retarded phonon-induced interaction between fermions:  $V_{ph}(\mathbf{k}, \omega) = \frac{-2g_{\mathbf{k}}^2 \omega_{\mathbf{k}}^0}{(\omega_{\mathbf{k}}^0)^2 - (\omega + i\delta)^2}$ . When the phonon velocity is smaller than the Fermi velocity as considered in this Letter, the dynamical screening due to fermion density fluctuations has to be included by calculating the Dyson's equations shown in Fig. 1(a), where  $U^\parallel = V_{ph}$  and  $U^\perp = V_{ph} + U_{ff}$  are the bare effective interaction between fermions in the spin parallel/perpendicular channels. The full dressed effective interaction can then be easily derived to be [11]:

$$V_{\text{eff}}^{\parallel,\perp} = \frac{(1 - U^{\parallel,\perp} P) U^{\parallel,\perp} + U^\perp P U^\perp}{(1 - U^\parallel P)^2 - (P U^\perp)^2} \quad (2)$$

where  $P(\mathbf{k}, \omega)$  is the polarizability due to fermionic quasi-particle and Cooper pair fluctuations. In Fig. 2 we show a numerical result of  $V_{\text{eff}}^\perp(q, \omega)$  and the screened phonon dispersion (obtained by tracking the peak position of  $\text{Im} V_{\text{eff}}^\perp(q, \omega)$ ) at  $T = T_c$  for a typical  $^{40}\text{K}$ - $^{87}\text{Rb}$  system [12]. As we will see later, although the screened phonon dispersion is close to the bare (unscreened) results, their spectral weights can still be quite different due to the fluctuations near the Fermi surface.

Now the single particle Green's function,  $G(\mathbf{p}, t) = -i\langle \hat{T} f_{\mathbf{p},\sigma}(t) f_{\mathbf{p},\sigma}^\dagger(0) \rangle$ , and the anomalous Green's func-



tion  $F(\mathbf{p}, t) = -i\langle \hat{T} f_{-\mathbf{p},\downarrow}(t) f_{\mathbf{p},\uparrow}(0) \rangle$  (where  $\hat{T}$  is the time-ordering operator) can be calculated within a self-consistent Hartree-Fock approximation as shown in Fig. 1(b). Such meanfield treatment is justified in strong fermion-phonon interaction regime because the vertex correction is proportional to the ratio of the phonon velocity to the Fermi velocity (known as Migdal's theorem [10]) and hence its leading order correction to the bare vertex (Fig. 1(d)) can be rather small ( $\Gamma_1 < 0.1$ , see Ref. [12]) in typical  $^{40}\text{K}$ - $^{87}\text{Rb}$  or  $^6\text{Li}$ - $^{23}\text{Na}$  mixtures. By the same reason, we can also approximate the full fermion polarizability,  $P$ , by a bubble diagram,  $P_0(\mathbf{k}, \omega) = \frac{2}{i\Omega} \sum_{\mathbf{p}} \int \frac{d\nu}{2\pi} G(\mathbf{p}, \nu) G(\mathbf{p} + \mathbf{k}, \nu + \omega)$ , without vertex lines inside the bubble diagram (Fig. 1(c)). The contribution coming from pair fluctuations can be neglected since here we are only interested in the calculation of  $T_c$ .

To formulate the Migdal-Eliashberg equations (MEE, see Fig. 1(b)) for a BFM system, we note that the phonon excitation energy,  $\omega_{\mathbf{k}}^0$ , is unbounded in the high energy limit, and hence the involved momentum integration cannot be restricted to the vicinity of the Fermi surface, as done in the original theory for the conventional superconductors [11]. We therefore define the following energy dependent interaction strength:

$$\widetilde{\alpha^2 F}_{\parallel,\perp}(\nu, E) \equiv \int_{FS} \frac{d^2 \mathbf{p} B_{\parallel,\perp}(\mathbf{p} - \sqrt{2m_f E} \hat{z}, \nu)}{v_F (2\pi)^3 N(E_F)/N(E)}, \quad (3)$$

where  $B_{\parallel,\perp}(\mathbf{q}, \nu) \equiv -\pi^{-1} \text{Im} V_{\text{eff}}^{\parallel,\perp}(\mathbf{q}, \nu)$  is the spectral weight of the dressed interaction;  $\int_{FS}$  denotes the integration over Fermi surface, where the gap function is measured.  $N(E) = m_f \sqrt{2m_f E}/2\pi^2$  is the 3D density of states for  $E > 0$ . Note that  $\widetilde{\alpha^2 F}_{\parallel,\perp}(\nu, E)$  drops to zero rapidly for  $E \rightarrow \infty$  due to the sharp spectral function,

$B_{\parallel,\perp}$ , near the phonon excitation energy (see Inset of Fig. 2). Following the same argument as Eliashberg [11], we can neglect the momentum dependence of self-energies,  $S$  and  $W$ , and then solve Fig. 1(b) by introducing the gap function,  $\Delta(\omega) \equiv W(\omega)/Z(\omega)$ , and a renormalization coefficient,  $Z(\omega) \equiv 1 - (S(\omega) - S(-\omega))/2\omega$  [11].

$Z(\omega \rightarrow 0)^{-1} \leq 1$  measures the single particle spectral weight at Fermi surface, and is equal to one for noninteracting fermions. Details of the derivation for a BFM system will be presented elsewhere [13]. The obtained MEE are the following:

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$$\omega(1 - Z(\omega)) = - \int_0^\infty d\nu \int_{-\infty}^\infty d\epsilon \widetilde{\alpha^2 F_{\parallel}}(\nu, \mu + \epsilon \tilde{Z}(\epsilon)) \text{Re} \left[ \frac{|\epsilon|}{\sqrt{\epsilon^2 - \Delta(\epsilon)^2}} \right] \left[ \frac{n_F(\epsilon) + n_B(\nu)}{\epsilon - \omega - \nu - i\delta} + \frac{1 - n_F(\epsilon) + n_B(\nu)}{\epsilon - \omega + \nu - i\delta} \right] \quad (4)$$

$$\begin{aligned} \Delta(\omega)Z(\omega) &= U_{ff} \int_{-\infty}^\infty d\epsilon n_F(\epsilon) \text{Re} \left[ \frac{\Delta(\epsilon) \text{sgn}(\epsilon)}{\sqrt{\epsilon^2 - \Delta(\epsilon)^2}} \right] N(\mu + \epsilon \tilde{Z}(\epsilon)) \\ &\quad + \int_0^\infty d\nu \int_{-\infty}^\infty d\epsilon \widetilde{\alpha^2 F_{\perp}}(\nu, \mu + \epsilon \tilde{Z}(\epsilon)) \text{Re} \left[ \frac{\Delta(\epsilon) \text{sgn}(\epsilon)}{\sqrt{\epsilon^2 - \Delta(\epsilon)^2}} \right] \left[ \frac{n_F(\epsilon) + n_B(\nu)}{\epsilon - \omega - \nu - i\delta} + \frac{1 - n_F(\epsilon) + n_B(\nu)}{\epsilon - \omega + \nu - i\delta} \right] \end{aligned} \quad (5)$$


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where  $n_{F/B}(\epsilon) = (e^{\epsilon/T} \pm 1)^{-1}$  is the Fermi/Bose distribution function.  $\tilde{Z}(\epsilon)$  is the real part of  $Z(\epsilon)\sqrt{1 - (\Delta(\epsilon)/\epsilon)^2}$ . Note that the instantaneous interaction,  $U_{ff}$ , contributes to the self-energy,  $S$ , via a constant shift only [11] and therefore does not appear in the r.h.s. of Eq. (4). Furthermore, the ultraviolet divergence caused by the pseudo-potential  $U_{ff}$  has also been removed in Eq. (5) by eliminating the bare Lippmann-Schwinger equations *in vacuo* as done in the weak coupling theory [2,4]. Therefore the MEE shown above have included both the non-retarded direct interaction and the retarded induced interaction on equal footing.

To calculate the  $T_c$  of Cooper pairs we first linearize the above equations by taking  $\Delta(\omega) \rightarrow 0$  and approximate  $Z(\omega)$  by  $Z_0 = Z(\omega \rightarrow 0)$ . Assuming  $T_c$  is much smaller than Fermi energy and typical phonon energy, we can simplify Eqs. (4)-(5) to be

$$1 - Z_0 = -2 \int_0^\infty d\nu \int_0^\infty d\epsilon \frac{\widetilde{\alpha^2 F_{\parallel}}(\nu, \mu + Z_0\epsilon)}{(\epsilon + \nu)^2} \quad (6)$$

$$\begin{aligned} Z_0 &= U_{ff} \int_{-\infty}^\infty \frac{d\epsilon}{\epsilon} n_F(\epsilon) N(\mu + \epsilon Z_0) \\ &\quad + \int_0^\infty d\nu \int_{-\infty}^\infty \frac{n_F(\epsilon) d\epsilon}{\epsilon(\epsilon - \nu)} \widetilde{\alpha^2 F_{\perp}}(\nu, \mu + Z_0\epsilon). \end{aligned} \quad (7)$$


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To solve  $T_c$  and  $Z_0$  analytically we further apply the single mode approximation,  $\text{Im}V_{\text{eff}}^{\parallel,\perp}(q, \nu) \sim -\pi V_{\text{im}}^{\parallel,\perp}(q) \delta(\nu - \omega_q)$ , where  $V_{\text{im}}^{\parallel,\perp}(q) \equiv -\frac{1}{\pi} \int_0^\infty d\nu \text{Im}V_{\text{eff}}^{\parallel,\perp}(q, \nu)$  is the total spectral weight. The phonon energy,  $\omega_q$ , is obtained from the peak position of  $\text{Im}V_{\text{eff}}^{\parallel,\perp}(q, \nu)$ . Within such approximation  $\widetilde{\alpha^2 F_{\parallel,\perp}}$  of Eq. (3) can be greatly simplified and  $T_c$  of Eq. (7) can be solved to be

$$\frac{T_c}{E_F} = \left( \frac{8\gamma}{\pi e^2 Z_0} \right) \exp \left[ \frac{1 + \lambda_{\parallel}(Z_0) + \lambda_{\log}(D)}{\lambda_0 - \lambda_{\perp}(Z_0)} \right], \quad (8)$$

where  $\gamma \sim 1.781$ .  $\lambda_0 \equiv N(E_F)U_{ff}$  and  $\lambda_{\perp}(Z_0) \equiv 2 \int_0^\infty \frac{d\nu}{\nu} \widetilde{\alpha^2 F_{\perp}}(\nu, 0)$  are respectively the direct and induced coupling strengths for Cooper pairs.  $D \equiv 4E_F/e^2 Z_0$  is an energy scale of Fermi energy.  $Z_0$  is then determined by solving  $Z_0 = 1 + \lambda_{\parallel}(Z_0)$  self-consistently since  $Z_0$  appears in the effective interaction via the polarizability,  $P$ . The two coupling strengths,  $\lambda_{\parallel}(Z_0)$  and  $\lambda_{\log}(D)$ , are given by (here  $\nu_c \equiv \omega_{2k_F}$ )

$$\lambda_{\parallel}(Z_0) = \int_0^{\nu_c} d\nu A^{\parallel}(\nu) \left[ \frac{2}{\nu} - \frac{1}{\nu + |\epsilon_{-}(\nu)|} - \frac{1}{\nu + \epsilon_{+}(\nu)} \right] - \int_{\nu_c}^\infty d\nu A^{\parallel}(\nu) \left[ \frac{1}{\nu + \epsilon_{-}(\nu)} - \frac{1}{\nu + \epsilon_{+}(\nu)} \right] \quad (9)$$

$$\lambda_{\log}(D) = \int_0^{\nu_c} \frac{d\nu A^{\perp}(\nu)}{\nu} \log \left( \frac{\nu^2 |\epsilon_{-}(\nu)| \epsilon_{+}(\nu)}{D^2 (\nu + |\epsilon_{-}(\nu)|)(\nu + \epsilon_{+}(\nu))} \right) + \int_{\nu_c}^\infty \frac{d\nu A^{\perp}(\nu)}{\nu} \log \left( \frac{\epsilon_{+}(\nu + \epsilon_{-}(\nu))}{\epsilon_{-}(\nu + \epsilon_{+}(\nu))} \right), \quad (10)$$


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where  $A^{\parallel,\perp}(\nu) \equiv \frac{N(E_F)}{2k_F^2} \left| \frac{\partial \omega_q(q_*)}{\partial q} \right|^{-1} q_* V_{\text{im}}^{\parallel,\perp}(q_*)$  with

$q_*(\nu)$  given by  $\omega_{q_*}(\nu) = \nu$ , and  $\epsilon_{\pm}(\nu) \equiv q_*^2/2m_f \pm$

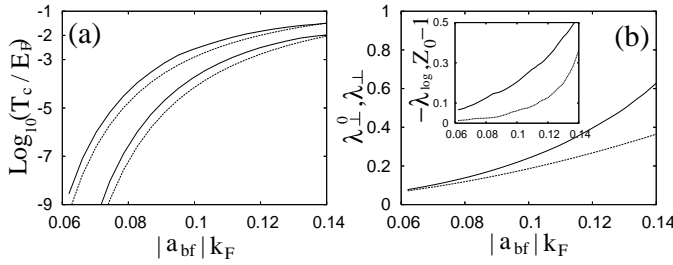


FIG. 3: (a) Superfluid temperature as a function of boson-fermion interaction strength,  $|a_{bf}|k_F$ , for the strong coupling (solid lines) and weak coupling (dashed lines) theories. Upper/lower pairs of lines are for boson density  $n_b = 100/50 \times \text{um}^{-3}$ . Other parameters are given in Ref. [12]. (b)  $\lambda_{\perp}$  and  $\lambda_{\parallel}^0$  ( $\lambda_{\parallel} = Z_0 - 1$  and  $-\lambda_{\log}$  for the inset) in the same calculation of (a) with  $n_b = 100 \text{ um}^{-3}$ .

$q_* k_F / m_f$ .

It is instructive to compare the analytical  $T_c$  in Eq. (8) with the weak coupling results [4]: one can see that Eq. (8) reproduces the weak coupling result [4]:  $T_c^0 = \frac{8\gamma}{\pi e^2} \exp\left[\frac{1}{\lambda_0 - \lambda_{\perp}^0}\right]$  if  $|\lambda_{\parallel}(Z_0)|, |\lambda_{\log}(D)| \ll 1$ . Here  $\lambda_{\perp}^0 = \frac{N(E_F)U_{bf}^2}{U_{bb}(2k_F\xi_b)^2} \ln(1 + (2k_F\xi_b)^2)$  is the phonon-induced coupling strength without any dynamical screening [4]. (Note that here we only discuss the  $s$ -wave pairing symmetry. The non  $s$ -wave scattering strength is in general very small and can be neglected.) In Fig. 3(a) we compare numerical results of  $T_c$  and  $T_c^0$ , using typical parameters of  $^{40}\text{K}$ - $^{87}\text{Rb}$  systems [12]. We find both of them increase sub-exponentially as the interaction strength,  $|a_{bf}|k_F$ , increases.  $T_c$  is larger than  $T_c^0$  by a factor of

2-8 for  $|a_{bf}|k_F < 0.12$ , and then becomes almost equal to  $T_c^0$  for stronger interaction. Note that  $T_c/T_c^0$  is large for small  $|a_{bf}|k_F$  simply because the repulsive direct interaction,  $U_{ff} > 0$ , sets the minimum value of  $|a_{bf}|k_F$  for pairing phase. This ratio will decrease to one as  $|a_{bf}|k_F \rightarrow 0$  if  $U_{ff} \leq 0$ . When  $|a_{bf}|k_F > 0.15$  the screened phonon velocity,  $c_b$ , becomes imaginary and the system becomes unstable toward collapse. This critical value of  $|a_{bf}|k_F$  is a little higher than the results predicted by the weak coupling theory [2] due to the reduction of the density of states in the strong coupling theory. Analyzing each quantity of Eq. (8) more carefully (see Fig. 3(b)), we find that although the ratio of  $\lambda_{\perp}$  to  $\lambda_{\parallel}^0$  increases as  $|a_{bf}|k_F$  increases via dynamical screening, the final ratio of  $T_c$  to  $T_c^0$  still becomes smaller due to the strong quasi-particle renormalization,  $Z_0$  (see the inset), in the strong coupling regime. Numerically solving the full strong coupling equations (Eqs. (4)-(5)) can also provide more information about the gap function spectroscopy,  $\Delta(\omega)$ , and fermion single particle spectral function etc., which can be measured by using rf-spectroscopy and/or Bragg scattering spectroscopy in the present experimental technique.

In summary, we have derived the full strong coupling theory for the fermion  $s$ -wave pairing phase in a Bose-Fermi mixture. Our results apply to the limit of slow phonon velocity and hence are valid in the existing BFM system ( $^{40}\text{K}$ - $^{87}\text{Rb}$  or  $^6\text{Li}$ - $^{23}\text{Na}$ ). The predicted critical temperature for a typical  $^{40}\text{K}$ - $^{87}\text{Rb}$  mixture can be as high as a few percents of the Fermi energy, which should be achievable by present experimental groups.

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- <sup>11</sup> For a review of strong coupling theory in superconductors, please see G.D. Mahan, *Many-Particle Physics* (Plenum press, New York, 1990), and D.J. Scalapino, in *Superconductivity*, ed. by R.D. Parks (Marcel Dekker, New York, 1969).
- <sup>12</sup> The parameters used in this Letter are for a typical  $^{40}\text{K}$ - $^{87}\text{Rb}$  system:  $^{40}\text{K}$  are prepared in the hyperfine states:  $|F, m_F\rangle = |9/2, -9/2\rangle$  and  $|9/2, -7/2\rangle$ .  $^{87}\text{Rb}$  are prepared in state  $|1, 1\rangle$ . Fermion density,  $n_f = 1 \text{ } \mu\text{m}^{-3}$ ; boson density  $n_b = 100 \text{ } \mu\text{m}^{-3}$ ; background  $s$ -wave scattering lengths (in unit of Bohr radius):  $a_{ff} = 174$ ,  $a_{bb} = 100$ , and  $a_{bf} = -330$ . Using these parameters, we have  $c_b/v_F = 0.6$ ,  $k_F\xi_b = 0.75$ , and the leading order vertex correction,  $\Gamma_1 = 0.07$ . See for example the experimental parameters of Ref. [8].
- <sup>13</sup> D.-W. Wang, in preparation.